How to fit transmission lines with ZFit

I – INTRODUCTION

ZFit is the impedance fitting tool of EC-Lab®. This note will describe how to fit transmission lines using one equivalent circuit elements contained in ZFit.

It has been well known for a long time that the Warburg impedance is equivalent to that of a semi-infinite large network i.e. a transmission line, as shown in Fig. 1 [1, 2].

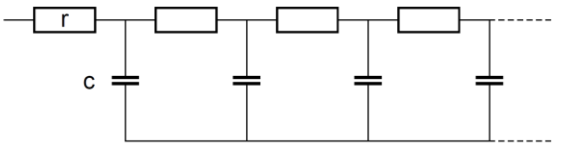

Figure 1: The equivalent circuit of the War-Burg impedance.

More recently it has been shown [3] that the impedance of a L-long transmission line made of χ and ζ elements and terminated by a Z_L element (Fig. 2) is given by the general expression:

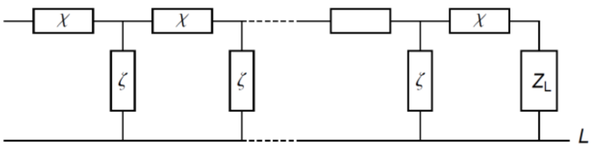

Figure 2: Uniform transmission line made χ and ζ elements and terminated by Z_L [3].

$$Z = \frac{\left(\zeta\chi - Z_L^2\right)\mathrm{sh}\left(\frac{L\sqrt{\chi}}{\sqrt{\zeta}}\right)}{Z_L\,\mathrm{sh}\left(\frac{L\sqrt{\chi}}{\sqrt{\zeta}}\right) + \sqrt{\zeta\chi}\,\mathrm{ch}\left(\frac{L\sqrt{\chi}}{\sqrt{\zeta}}\right)} + Z_L$$

With three limiting cases

- open-circuited transmission line

$$Z_L = \infty \Rightarrow Z = \sqrt{\zeta\chi}\,\coth\left(\frac{L\sqrt{\chi}}{\sqrt{\zeta}}\right) \qquad (1)$$

- short-circuited transmission line

$$Z_L = 0 \Rightarrow Z = \sqrt{\zeta\chi}\,\mathrm{th}\left(\frac{L\sqrt{\chi}}{\sqrt{\zeta}}\right) \qquad (2)$$

- semi-infinite transmission line

$$L \to \infty \Rightarrow Z = \sqrt{\zeta\chi} \qquad (3)$$

Hereafter, some transmission lines are described and the corresponding ”simple” equivalent circuit elements are shown. Firstly, the open-circuited transmission lines will be explained, followed by short-circuited and semi-infinite transmission lines[1].

III – OPEN-CIRCUITED TRANSMISSION LINES Z_L = 1

II - 1 OPEN-CIRCUITED URC (UNIFORM DISTRIBUTED RC)

Let us consider the open-circuited tranmission line made of r and c elements (Fig. 3).

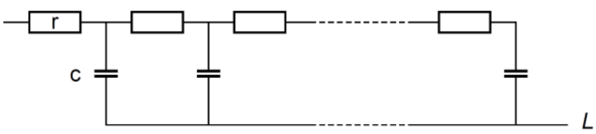

Figure 3: L-long open uniform distributed RC (URC) transmission line [4,5].

[1] The transmission lines are named accordingly to the U-χζ format where U means uniformly distributed and χ and ζ are the elements of the transmission line.

Using Eq. (1), the transmission line impedance is given by:

$$\chi = r, \zeta = \frac{1}{j\omega c} \Rightarrow Z = \sqrt{r}\frac{\coth\left(L\sqrt{rcj\omega}\right)}{\sqrt{cj\omega}} \quad (4)$$

With $\omega = 2\pi f$. This impedance is similar to that of the M element of ZFit

$$Z_M = R_d \frac{\coth\sqrt{\tau_d j\omega}}{\sqrt{\tau_d j\omega}}, R_d = L_r, \tau_d = L^2 rc \quad (5)$$

II - 2 OPEN-CIRCUITED URQ

Replacing c elements by q elements, with $Z_q = 1\big/\left(q\left(j\omega\right)^{\alpha}\right)$ leads to transmission line shown in Fig. 4.

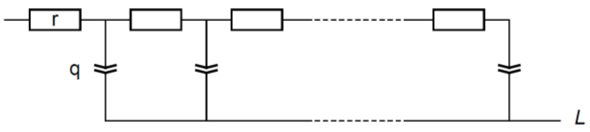

Figure 4: L-long open uniform distributed RQ|(URQ) transmission line.

The transmission line impedance is given by

$$\chi = r, \zeta = \frac{1}{q\left(j\omega\right)^{\alpha}}$$

$$\Rightarrow Z = \sqrt{r}\frac{\coth\left(L\sqrt{rq}\left(j\omega\right)^{\alpha/2}\right)}{\sqrt{q}\left(j\omega\right)^{\alpha/2}} \quad (6)$$

The impedance is similar to that of the M_a element of ZFit.

$$Z_{M_a} = R\frac{\coth\left(\tau j\omega\right)^{\alpha/2}}{\left(\tau j\omega\right)^{\alpha/2}} \quad (7)$$

With $R = Lr, \tau = \left(L^2 rq\right)^{1/\alpha}$

As an example a Nyquist impedance diagram of a battery Ni-MH 1900 mAh is shown in Fig. 5. The equivalent circuit R1+L1+Q1/(R2+Ma3), containing a M_a element, is chosen to fit the data shown in Fig. 5. The values of the parameters, obtained using the ZFit tool of EC-Lab, are $R1$ = 0.049 Ω, $L1$ = 0.154 10^{-6} H, $Q1$ = 0.66 F $s^{\alpha-1}$, $\alpha 1$ = 0.61, $R2$ = 0.0236 Ω, $R3 = L\ r$ = 0.057 Ω, $\tau 3 = (L^2\ r\ q)^{1/\alpha}$ = 2.25 s and $\alpha 3$ = 0.89.

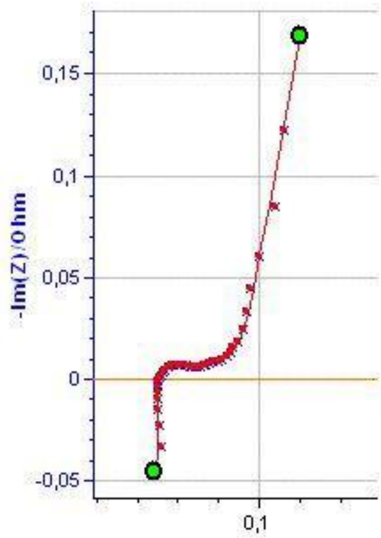

Figure 5: Nyquist impedance diagram of a battery Ni-MH 1900 mAh.

The equivalent circuit of the so-called anomalous diffusion is shown in Fig. 6 [6].

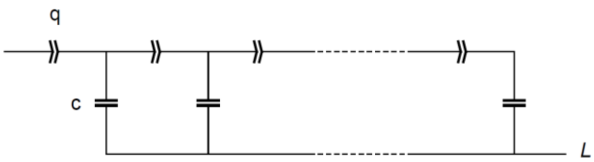

Figure 6: L-long open uniform distributed QC (UQC) transmission line. Anomalous diffusion [6].

The anomalous diffusion impedance is given by

$$\chi = \frac{1}{q(j\omega)^{\alpha}}, \zeta = \frac{1}{cj\omega}$$

$$\Rightarrow Z = \frac{\coth\left(L\sqrt{\frac{c}{q}}(j\omega)^{\frac{1}{2}-\frac{\alpha}{2}}\right)}{\sqrt{cq}(j\omega)^{\frac{\alpha}{2}+\frac{1}{2}}} \quad (8)$$

This impedance is similar to that of the M_g element of ZFit

$$Z_{M_g} = R\frac{\coth(\tau j\omega)^{\gamma/2}}{(\tau j\omega)^{1-\gamma/2}} \quad (9)$$

With $\gamma = 1-\alpha$, $R = c^{\frac{1}{\gamma}-1} L^{\frac{2}{\gamma}-1} q^{\frac{-1}{\gamma}}, \tau = c^{\frac{1}{\gamma}} L^{\frac{2}{\gamma}} q^{\frac{-1}{\gamma}}$.

III – SHORT-CIRCUITED TRANSMISSION LINES Z_L=0

III - 1 SHORT-CIRCUITED URC

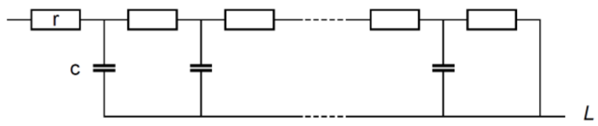

Figure 7: L-long open uniform distributed QC (UQC) transmission line. Anomalous diffusion [6].

Using Eq. (2), the impedance of the short-circuited transmission line made of r and c elements (Fig. 7) is given by

$$\chi = r, \zeta = \frac{1}{cj\omega}$$

$$\Rightarrow Z = r\frac{\mathrm{th}\left(L\sqrt{rcj\omega}\right)}{\sqrt{rcj\omega}} \quad (10)$$

This impedance is similar to that og the W_d element of ZFit

$$Z_{W_d} = R_d \frac{\mathrm{th}\sqrt{\tau_d j\omega}}{\sqrt{\tau_d j\omega}}, R_d = Lr, \tau_d = L^2 rc \quad (11)$$

IV – SEMI-INFINITE TRANSMISSION LINES: $L \to \infty$

IV - 1 SEMI-INFINITE URC

The impedance of the semi-infinite transmission line shown in Fig. 1 is obtained making $L \to \infty$ in Eq. (10).

$$L \to \infty \Rightarrow Z = r\frac{\mathrm{th}\left(L\sqrt{rcj\omega}\right)}{\sqrt{\sqrt{rcj\omega}}} \approx \frac{\sqrt{r}}{\sqrt{cj\omega}} \quad (12)$$

This expression is similar to that of the Warburg (W) element of ZFit

$$Z_W = \frac{2\sigma}{\sqrt{j\omega}} \text{ with } \sigma = \frac{\sqrt{r}}{2\sqrt{c}} \quad (13)$$

As an example a Nyquist impedance diagram of a Fe(II)/Fe(III) system is shown in Fig. 8.

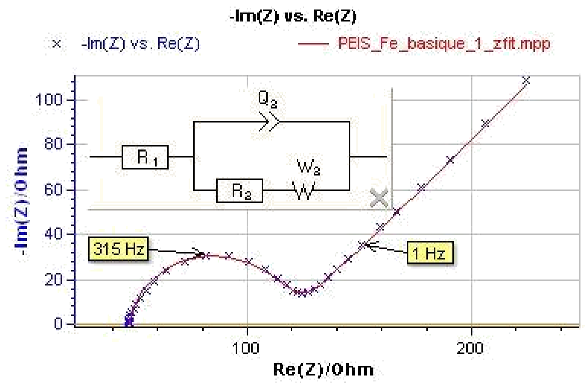

Figure 8: Nyquist impedance diagram of a Fe(III)/Fe(II) system in basic medium.

The Randles circuit R1+Q2/(R2+W2), containing a Warburg element, is chosen to fit the data shown in Fig. 8. The values of the parameters for equivalent circuit are R_1 = 47.57 Ω, Q_2 = 17.09 x 10^{-6} F s^{-1}, α = 0.885, R_2 = 70.94 Ω and σ_2 = 85.33 Ω $s^{-1/2}$

$$\Rightarrow \sqrt{\frac{r}{c}} = 42.7 \ \Omega \ s^{-1/2}.$$

IV - 2 SEMI-INFINITE URRC

First of all, let us calculate the impedance of the L-long URRC transmission line (Fig. 9) corresponding to diffusion-reaction and diffusion-trapping impedance [7]:

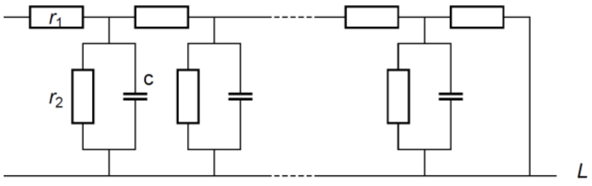

Figure 9: L-long short-circuited uniform distributed RRC (URRC) transmission line.

$$\chi = r_1, \zeta = \frac{r_2}{1 + r_2 c j\omega}$$

$$\Rightarrow Z = \sqrt{r_1 r_2}\, \frac{\operatorname{th}\left(L\sqrt{\frac{r_1}{r_2}\left(1 + r_2 c j\omega\right)}\right)}{\sqrt{1 + r_2 c j\omega}} \quad (14)$$

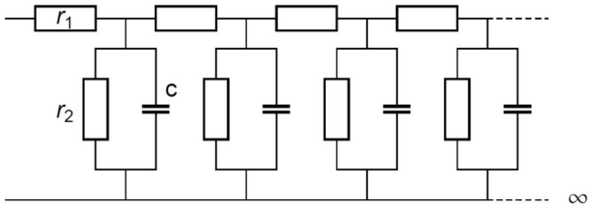

Figure 10: Semi-infinite short-circuited uniform distributed RRC (URRC) transmission line.

With $L \to \infty$ it is obtained [8]:

$$L \to \infty \Rightarrow Z \approx \frac{\sqrt{r_1 r_2}}{\sqrt{1 + r_2 c j\omega}} \quad (15)$$

This expression is similar to that of the Gerischer element G of ZFit [9]:

$$Z_G = \frac{R_G}{\sqrt{1 + \tau_G j\omega}}, R_G = \sqrt{r_1 r_2}, \tau_G = r_2 c \quad (16)$$

IV - 3 SEMI-INFINITE URRQ

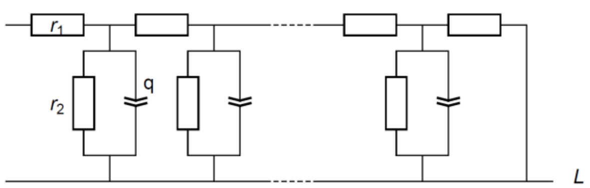

Figure 11: L-long short-circuited uniform distributed RRQ (URRQ) transmission line.

Replacing c elements by q elements

$$\chi = r_1, \zeta = \frac{r_2}{1 + r_2 c \left(j\omega \right)^{\alpha}}$$

$$\Rightarrow Z = \sqrt{r_1 r_2}\, \frac{\operatorname{th}\left(L\sqrt{\frac{r_1}{r_2}\left(1 + r_2 c \left(j\omega \right)^{\alpha}\right)}\right)}{\sqrt{1 + r_2 c \left(j\omega \right)^{\alpha}}} \quad (17)$$

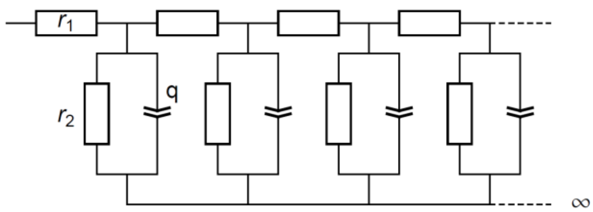

Figure 12: Semi-infinite short-circuited uniform distributed RRQ (URRQ) transmission line.

And $L \to \infty \Rightarrow Z \approx \frac{\sqrt{r_1 r_2}}{\sqrt{1 + \tau \left(j\omega \right)^{\alpha}}}$ (18)

This expression is similar to that of the G_a element of ZFit

$$Z_{G_a} = \frac{R}{\sqrt{1 + \tau \left(j\omega \right)^{\alpha}}}, R = \sqrt{r_1 r_2}, \tau = r_2 q \quad (19)$$

V – CONCLUSION

Seven elements, W, Wd, M, Ma, Mg, G and Ga, available in ZFit correspond to different transmission lines (Tabs. I and II).

Table I: Summary table.

Transmission line		ZFit Element
Open Circuited	URC	M
	URQ	Ma
	UQC	Mg
Short circuited	URC	Wd
Semi-infinite	URC	W
	URRC	G
	URRQ	Ga

Data files can be found in :
C:\Users\xxx\Documents\EC-Lab\Data\Samples\EIS\PEIS_Fe_Basique_1

and AN43_peis_batteries_carouf_01_PEIS_C06

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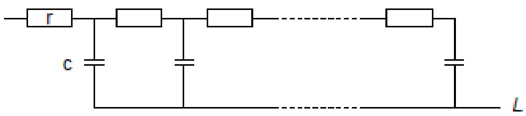
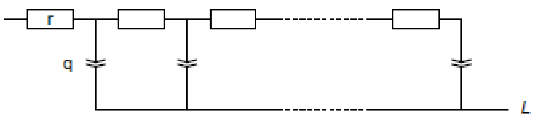
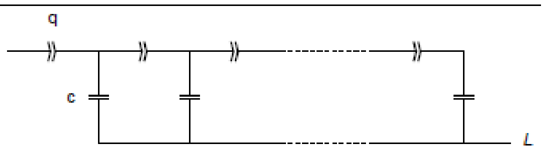
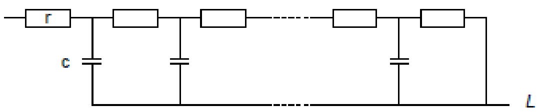
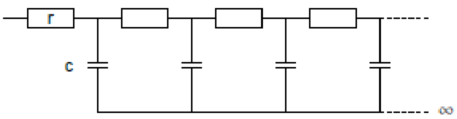
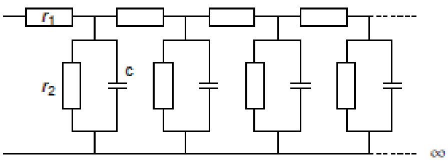
Table II: ZFit elements *vs.* transmission lines

ZFit element	Equations	Transmission line
M	$R_d \frac{\coth\sqrt{\tau_d\, j\,\omega}}{\sqrt{\tau_d\, j\,\omega}}$ $R_d = L\, r,\ \ \tau_d = L^2\, r\, c$	
M_a	$R \frac{\coth(\tau\, j\,\omega)^{\alpha/2}}{(\tau\, j\,\omega)^{\alpha/2}}$ $R = L\, r$ $\tau = (L^2\, r\, q)^{1/\alpha}$	
M_g	$R \frac{\coth(\tau\, j\,\omega)^{\gamma/2}}{(\tau\, j\,\omega)^{1-\gamma/2}}$ $R = c^{\frac{1}{\gamma}-1} L^{\frac{2}{\gamma}-1} q^{-1/\gamma}$ $\tau = c^{\frac{1}{\gamma}} L^{2/\gamma} q^{-1/\gamma}$	
W_d	$R_d \frac{\text{th}\sqrt{\tau_d\, j\,\omega}}{\sqrt{\tau_d\, j\,\omega}}$ $R_d = L\, r$ $\tau_d = L^2\, r\, c$	
W	$\frac{2\,\sigma}{\sqrt{j\,\omega}}$ $\sigma = \frac{\sqrt{r}}{2\,\sqrt{c}}$	
G	$\frac{R_G}{\sqrt{1+\tau_G\, j\,\omega}}$ $R_G = \sqrt{r_1\, r_2}$ $\tau_G = r_2\, c$	
G_a	$\frac{R_G}{\sqrt{1+\tau_G\, (j\,\omega)^{\alpha}}}$ $R_G = \sqrt{r_1\, r_2}$ $\tau_G = r_2\, q$