How to read EIS accuracy contour plots

I Introduction

The impedance measurement accuracy of an impedance analyser depends on the impedance magnitude and on the measurement frequency. Potentiostats/galvanostats/EIS (Electrochemical Impedance Spectroscopy) specifications are given in graphs called accuracy contour plots (1). The aim of this note is to help the user understand and use the EIS accuracy contour plots.

II Contour plot

Fig. 1: 3D graph of a two-variable function, level curve (blue curve) and projection in the \( x, y \) plan (red curve).

Contour plots are isovalue curves of a function depending on two variables. As an example 3D graph of a two variables function \( f(x, y) = x^2 + y^2 \) and a level curve of this function for \( f(x, y) = 10 \) are shown in Fig. 1. The level curve is a cross-section of the 3D graph parallel to the \( (x, y) \) plan at \( f(x, y) = 10 \). The projection of this curve on the \( x, y \) plan is a contour plot. For any point \( (x, y) \) inside of this contour, we have \( f(x, y) < 10 \) and for any point outside of this contour we have \( f(x, y) \geq 10 \).

III EIS accuracy contour plot

III.1 SP-300 EIS accuracy contour plot

The scheme of an EIS accuracy contour plot is shown in Fig. 2.

Fig. 2: Scheme of an EIS accuracy contour plot.

The two horizontal limits (high (5) and low (1) \( Z \)) are related to the ability of the instrument to measure/control accurately the current. To increase the high \( Z \) portion of accuracy contour plot, the instrument has to be able to measure low current (often related to the electrometer impedance in the specifications).

1 Considering the definitions given in [1] and recalled in Appendix A, the word accuracy is a time-honored misnomer. Uncertainty should preferably be used.
To increase the low portion of the accuracy contour plot, the instrument has to be able to manage high current. Current booster may be added to improve the measurement in this area. The vertical limit (3) depends on the regulation speed of the instrument. This is the maximum frequency that can be reached by the instrument with an acceptable error. It is related to the bandwidth of the instrument. The two other limits (4) and (2) are related to stray capacitor and stray inductor, respectively. Inductive effects are very sensitive to the current level and the cell connection.

The contour plot of an SP-300 potentiostat is shown in Fig. 3. The shaded areas show different ranges of impedances that can be measured at various frequencies within a specified error in magnitude and in phase. These errors refer to the borders and not to the areas as is explained below.

**III.2 Example**

Let us consider the black dot in Fig. 3. This dot corresponds to the measurement of an impedance $|Z| = 10^4 \ \Omega$ measured at a frequency $f = 10^6 \ \text{Hz}$. As this measurement is located between the 0.3 %, 0.3° and 1 %, 1° contour plots, the accuracy is 0.3 % $< \Delta |Z| / |Z| < 1 \%$, for the modulus (relative error) and $0.3^\circ < \Delta \phi_Z < 1^\circ$ for the phase (absolute error). The phase is given in absolute error because the relative error could be indeterminate in the case where $\phi = 0$. An impedance whose modulus is 100 $\Omega$ at $10^4 \ \text{Hz}$ is measured with a relative uncertainty lower than 0.3 % for the modulus and lower than 0.3° for the phase (Fig. 3, white dot).

**IV How to use an EIS contour plot**

**IV.1 Impedance of the $R_1 + R_2 / C_2$ circuit**

The impedance for the $R_1 + R_2 / C_2$ is given by

$$Z(f) = \frac{R_2}{1 + R_2 \ C_2 \ \omega^2} \quad (1)$$

The modulus of the impedance as a function of the frequency is shown in Fig. 4 for $R_1 = 0.1 \ \Omega$, $R_2 = 3 \ \Omega$ and $C_2 = 10^{-6} \ \text{F}$.

**Fig. 3: SP-300 EIS accuracy contour plot.**

Black dot: $f = 10^6 \ \text{Hz}, \ |Z| = 10^4 \ \Omega$, white dot: $f = 10^4 \ \text{Hz}, \ |Z| = 10^2 \ \Omega$.

**Fig. 4: Modulus Bode diagram of the impedance for the $R_1 + R_2 / C_2$ circuit.** $R_1 = 0.1 \ \Omega$, $R_2 = 3 \ \Omega$ and $C_2 = 10^{-6} \ \text{F}$. 
IV.2 Example #1

The modulus Bode impedance diagram of this electrical circuit $R_1 + R_2/C_2$ and the SP-300 contour plot are superimposed in Fig. 5. This figure allows to predict the precision of the impedance measurements as a function of the frequency. One has to determine the frequency values where the modulus diagram moves into a different precision domain. For instance, the modulus diagram moves from the high precision domain ($|\Delta Z|/|Z| < 0.3\%$ and $\Delta \phi Z < 0.3^\circ$) to the mid-range precision domain ($0.3 < |\Delta Z|/|Z| < 1\%$ et $0.3 < \Delta \phi Z < 1^\circ$) for a frequency moving from $f_1 \approx 10$ kHz to $f_2 \approx 60$ kHz (Fig. 5).

Fig. 5: Superimposition of the modulus Bode impedance diagram of the electrical circuit $R_1 + R_2/C_2$ and the SP-300 contour plot.

The Nyquist impedance diagram of the $R_1 + R_2/C_2$ circuit is show in Fig. 6 with the impedances measured at $f_1$, $f_2$ and $f_3$.

Fig. 6: Nyquist diagram of the impedance for the $R_1 + R_2/C_2$ circuit, $f_1 \approx 10$ kHz, $f_2 \approx 60$ kHz, $f_3 \approx 100$ kHz.

Tab. I shows how the precision of the impedance modulus and phase evolves with the frequency for the $R_1 + R_2/C_2$ electrical circuit with intermediate $R_s$ and $C$ values.

Tab. I: Impedance measurement accuracy as a function of the frequency. $R_1 = 0.1 \, \Omega$, $R_2 = 3 \, \Omega$ and $C_2 = 10^{-6} \, \text{F}$.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Impedance modulus</th>
<th>Impedance phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f &lt; f_1$</td>
<td>$\Delta</td>
<td>Z</td>
</tr>
<tr>
<td>$f_1 &lt; f &lt; f_2$</td>
<td>$0.3% &lt; \Delta</td>
<td>Z</td>
</tr>
<tr>
<td>$f_2 &lt; f &lt; f_3$</td>
<td>$1% &lt; \Delta</td>
<td>Z</td>
</tr>
<tr>
<td>$f &gt; f_3$</td>
<td>$\Delta</td>
<td>Z</td>
</tr>
</tbody>
</table>

IV.3 Example #2

Another situation can be observed for high values of $R_2$ and low values of $C_2$, for instance $R_1 = 10^4 \, \Omega$, $R_2 = 0.2 \times 10^7 \, \Omega$ and $C_2 = 5 \times 10^{-9} \, \text{F}$ (Figs. 7 and 8).

Fig. 7: Superimposition of the modulus Bode impedance diagram of the electrical circuit $R_1 + R_2/C_2$ (high $R_s$ values and low $C$ value) and the SP-300 contour plot.
\[ \text{Re} \frac{Z}{\Omega} / \text{Im} \frac{Z}{\Omega} \]

Fig. 8: Nyquist diagram of the \( R_1 + R_2/C_2 \) circuit. \( R_1 = 10^4 \, \Omega, \) \( R_2 = 0.2 \times 10^7 \, \Omega \) and \( C_2 = 5 \times 10^{-10} \, \text{F}. \)

As the frequency reaches lower values, the measurement precision moves outside of the best precision domain. Tab. II shows how the precision of the impedance modulus and phase evolves with the frequency for an \( R + R/C \) electrical circuit with high \( R \) values and low \( C \) values.

Tab. II: Change of impedance measurement accuracy with frequency. \( R_1 = 10^4 \, \Omega, \) \( R_2 = 0.2 \times 10^7 \, \Omega \) and \( C_2 = 5 \times 10^{-10} \, \text{F}. \)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Impedance modulus</th>
<th>Impedance phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f &lt; f_1 )</td>
<td>( 0.3 % &lt; \Delta</td>
<td>Z</td>
</tr>
<tr>
<td>( f_1 &lt; f &lt; f_2 )</td>
<td>( \Delta</td>
<td>Z</td>
</tr>
<tr>
<td>( f_2 &lt; f &lt; f_3 )</td>
<td>( 1 % &lt; \Delta</td>
<td>Z</td>
</tr>
<tr>
<td>( f &gt; f_4 )</td>
<td>( \Delta</td>
<td>Z</td>
</tr>
</tbody>
</table>

Interactive contour plots are shown on the Bio-Logic website at the URL: http://www.bio-logic.info/potentiostat-electrochemistry-ec-lab/apps-literature/interactive-eis/eis-accuracy-contour-plot/ (Fig. 9).

V Conclusion

This note aims at explaining how to read and understand EIS contour plots, which are provided with each of our instrument. Contour plots must be used to interpret errors made during an EIS measurement and to find the best frequencies to be used for a given impedance range.

A Definitions [1]

Measurand
Quantity intended to be measured, quantity subject to measurement.

Measurement accuracy
Closeness of agreement between a measured quantity value and a true quantity value of a measurand. The concept ‘measurement accuracy’ is not a quantity and is not given a numerical quantity value. A measurement is said to be more accurate when it offers a smaller measurement error.

Measurement uncertainty, uncertainty
Non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand.

Measurement precision, precision
Closeness of agreement between indications or measured quantity values obtained by replicate measurements on the same or similar objects under specified conditions.

References

How to use EIS contour plot.

Potentiostat SP-300

Simulation for the equivalent circuit

R1+R2/C2

Frequency parameter
log(\theta_{1u})
-1.

R1+R2/C2, common parameters
log(R1;\Omega)
-1.
log(R2;\Omega)
0.5
log(C2;F)
-5.

L1=R1+R2/C2
log(L1;H)
-6.

R1+C2/(R2+M2)
log(R1;\Omega)
-1.

\text{Fig. 9: Example of an interactive EIS accuracy contour plot.}